

Non-abelian black strings and cosmological constant

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Abstract

We study the classical solutions of the Einstein-Yang-Mills model in five dimensions in the presence of a cosmological constant Λ . Using a spherically symmetric ansatz and assuming that the fields do not depend on the extra dimension, we transform the equations into a set of differential equations that we solve numerically. We construct new types of regular (resp. black holes) solutions which, close to the origin (resp. the event horizon) resemble the 4-dimensional gravitating monopole (resp. non abelian black hole) and study their global properties.

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I. INTRODUCTION

It is believed that topological defects have occurred and played a role during some phase transitions in the evolution of the Universe, see e.g. [1]. In particular, magnetic monopoles [2] must have been produced during the GUT symmetry breaking phase transition. The actual non-observation of them leads to constraints which have to be implemented in the models of inflation. On the other hand observational evidences performed in the last years [3] favours the possibility that space-time has an accelerated expansion which could be related to a positive cosmological constant.

It is therefore natural to examine the properties of the various topological defects in the presence of a cosmological constant, or said in other words, in asymptotically DeSitter space-time. Recently [4] the magnetic monopole and the sphalerons occurring in an $SU(2)$ gauge theory spontaneously broken by a scalar potential were constructed in an asymptotically DeSitter space-time and it was found that the asymptotic decay of the matter field is not compatible with a finite mass.

On the other hand, the last years have seen an increase of attention for space-times involving more than four dimensions and of brane-world models to describe space, time and matter. The so-called brane-world models [5] have gained a lot of interest. These assume the standard model fields to be confined on a 3-brane embedded in a higher dimensional manifold.

A large number of higher dimensional black holes has been studied in recent years. The first solutions that have been constructed are the hyperspherical generalisations of well-known black holes solutions such as the Schwarzschild and Reissner-Nordström solutions in more than four dimensions [6] as well as the higher dimensional Kerr solutions [7]. In d dimensions, these solutions have horizon topology S^{d-2} .

However, in contrast to 4 dimensions black holes with different horizon topologies should be possible in higher dimensions. An example is a 4-dimensional Schwarzschild black hole extended into one extra dimension, a so-called Schwarzschild black string. These solutions have been discussed extensively especially with view to their stability [8]. A second example, which is important due to its implications for uniqueness conjectures for black holes in higher dimensions, is the black ring solution in 5 dimensions with horizon topology $S^2 \times S^1$ [9].

The by far largest number of higher dimensional black hole solutions constructed so far are

solutions of the vacuum Einstein equations, respectively Einstein-Maxwell equations. The first example of black hole solutions containing non-abelian gauge fields have been discussed in [10]. These are non-abelian black holes solutions of a generalised 5-dimensional Einstein-Yang-Mills system with horizon topology S^3 . Using ideas of [11, 12], SU(2)-black strings with $S_2 \times S_1$ topology were constructed in [13]. Several regular and black hole solutions of an Einstein-Yang-Mills model have been constructed recently with different symmetries [14, 15, 16]. These solutions are non-abelian black hole solutions in 4 dimensions extended into one extra codimension.

In this paper, we consider the Einstein-Yang-Mills lagrangian in five dimensions and gauge group SU(2). Along with [11] we make the hypothesis that the fields do not depend of the extra dimension and assume the fields to be spherically symmetric in standard three-dimensional space. As shown in [11, 12] the lagrangian can be dimensionally reduced to a four-dimensional effective lagrangian describing Yang-Mills-Higgs-dilaton (the Higgs field being a triplet of SU(2)) interacting minimally with the metric fields. As so the classical equations admit solutions similar to the gravitating monopole and monopole black holes [17], the dilaton plays however a major role and the pattern of classical solutions is considerably different.

If we extend the original model by means of a 5-dimensional cosmological constant, the reduced effective action is supplemented by a Liouville potential in the dilaton field [18]. Several explicit solutions of these equations have been constructed in the last years [18, 19] but, as far as we know, these solutions present singularities at the origin and/or are characterized by non-conventional asymptotic forms of the metric fields. Very recently, a detailed analysis of black string with negative cosmological constant was reported in [20]. Here we reconsider the classical equations in presence of a positive cosmological constant, insisting that, close to the origin (resp. to the event horizon), the fields behave like a gravitating monopole (resp. a black string). Such solutions are indeed constructed numerically with the appropriated boundary conditions and their properties are analyzed for different values of the coupling constants.

We give the model including the ansatz, equations of motion and boundary conditions in Section II. The numerical results corresponding to solution regular at the origin and solutions presenting an event horizon are discussed respectively in Sections III and IV. The summary is given in Section V.

II. THE MODEL

The Einstein-Yang-Mills Lagrangian in $d = (4 + 1)$ dimensions is given by:

$$S = \int \left(\frac{1}{16\pi G_5} (R - 2\Lambda_5) - \frac{1}{4e^2} F_{MN}^a F^{aMN} \right) \sqrt{g^{(5)}} d^5x \quad (1)$$

with the SU(2) Yang-Mills field strengths $F_{MN}^a = \partial_M A_N^a - \partial_N A_M^a + \epsilon_{abc} A_M^b A_N^c$, the gauge index $a = 1, 2, 3$ and the space-time index $M = 0, \dots, 5$. G_5 , Λ_5 and e denote respectively the 5-dimensional Newton's and cosmological constants and the coupling constant of the gauge field theory. G_5 is related to the Planck mass M_{pl} by $G_5 = M_{pl}^{-3}$ and e^2 has the dimension of [length].

Along with [11] we assume the metric and the matter fields to be independent on the extra coordinate y , the fields can be parametrized as follows :

$$g_{MN}^{(5)} dx^M dx^N = e^{-\xi} g_{\mu\nu}^{(4)} dx^\mu dx^\nu + e^{2\xi} dy^2, \quad \mu, \nu = 0, 1, 2, 3 \quad (2)$$

and

$$A_M^a dx^M = A_\mu^a dx^\mu + \Phi^a dy. \quad (3)$$

$g^{(4)}$ is the 4-dimensional metric tensor. After dimensional reduction, the lagrangian above leads to an effective 4-dimensional Einstein-Yang-Mills-Higgs-dilaton lagrangian whose matter part L_M is reads :

$$L_M = -\frac{1}{4} e^\xi F_{\mu\nu}^a F^{a,\mu\nu} - \frac{1}{2} e^{-2\xi} D_\mu \Phi^a D^\mu \Phi^a - \frac{3}{4} \frac{1}{\alpha^2} \partial_\mu \xi \partial^\mu \xi - \frac{\Lambda}{2} e^{-\xi}, \quad \alpha^2 \equiv 4\pi G \quad (4)$$

The cosmological constant in $d = 5$ leads to a Liouville potential for the dilaton [18] in $d = 4$.

A. The Ansatz

Our aim is to construct non-abelian regular and black strings solutions which are spherically symmetric in the four-dimensional space-time and are extended into one extra dimension. The topology of these non-abelian black strings will thus be $S^2 \times \mathbb{R}$ or $S^2 \times S^1$ if the extra coordinate y is chosen to be periodic.

For the metric the spherically symmetric Ansatz reads:

$$g_{MN}^{(5)} dx^M dx^N = e^{-\xi} [-A^2 N dt^2 + N^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d^2\varphi] + e^{2\xi} dy^2, \quad (5)$$

where N, A, ξ are function of the coordinate r only. As usual we define

$$N(r) = 1 - \frac{2m(r)}{r} - \frac{\Lambda}{6}r^2 \quad \text{or} \quad N(r) = 1 - \frac{2\tilde{m}(r)}{r} . \quad (6)$$

In these coordinates, $m(r)$ represents the (dimensionful) mass per unit length (of the extra dimension) of the solution.

For the gauge fields, we use the spherically symmetric ansatz [2] :

$$A_r^a = A_t^a = 0 , \quad (7)$$

$$A_\theta^a = (1 - K(r))e_\varphi^a , \quad A_\varphi^a = -(1 - K(r))\sin\theta e_\theta^a , \quad (8)$$

$$\Phi^a = vH(r)e_r^a , \quad (9)$$

where v is a mass scale.

B. Equations of motion

With the following rescalings:

$$x = evr , \quad \mu = evm \quad (10)$$

the resulting set of ordinary differential equations only depends on the fundamental coupling $\alpha \equiv 4\pi\sqrt{G_5}v$ and of the reduced cosmological constant $\Lambda \equiv 2\alpha^2\Lambda_5$.

The Einstein equations for the metric functions N, A and ξ then read:

$$\begin{aligned} \mu' = & \alpha^2 \left(e^\xi N(K')^2 + \frac{1}{2}Nx^2(H')^2 e^{-2\xi} + \frac{1}{2x^2}(K^2 - 1)^2 e^\xi + K^2 H^2 e^{-2\xi} \right) \\ & + \frac{3}{8}Nx^2(\xi')^2 + \frac{\Lambda}{4}x^2 e^{-\xi} , \end{aligned} \quad (11)$$

$$A' = \alpha^2 x A \left(\frac{2(K')^2}{x^2} e^\xi + e^{-2\xi}(H')^2 \right) + \frac{3}{4}x A (\xi')^2 , \quad (12)$$

$$\begin{aligned} (x^2 A N \xi')' = & \frac{4}{3}\alpha^2 A \left[e^\xi \left(N(K')^2 + \frac{(K^2 - 1)^2}{2x^2} \right) - 2e^{-2\xi} \left(\frac{1}{2}N(H')^2 x^2 + H^2 K^2 \right) \right] \\ & - \frac{\Lambda}{3}A x^2 e^{-\xi} , \end{aligned} \quad (13)$$

while the Euler-Lagrange equations for the matter functions K and H are given by:

$$(e^\xi A N K')' = A \left(e^\xi \frac{K(K^2 - 1)}{x^2} + e^{-2\xi} H^2 K \right) , \quad (14)$$

$$(e^{-2\xi}x^2ANH')' = 2e^{-2\xi}K^2AH , \quad (15)$$

where the prime denotes the derivative with respect to x .

These equations are invariant under the dilatation transform

$$e^\xi \rightarrow \lambda e^\xi , \quad x \rightarrow \lambda^{3/2}x , \quad \alpha^2 \rightarrow \lambda^2\alpha^2 , \quad \Lambda \rightarrow \lambda^{-2}\Lambda , \quad \mu \rightarrow \lambda^{3/2}\mu \quad (16)$$

where λ is an arbitrary constant. In [12] the "gauge" $\xi(\infty) = 0$ was chosen. Here, the symmetry above will be fixed by imposing $\xi(0) = 0$. The pattern of solutions is quite different with this choice, in particular solutions exist for arbitrary values of α as indicated on Fig. 1 which has to be contrasted with Figs.1 and 3 of [12]. In order to avoid confusion with previous papers on the topic, we will use α, x_h to denote the gravitation coupling constant and the black hole horizon in the gauge $\xi(\infty) = 0$ and α', x'_h to denote these quantities in the gauge $\xi(0) = 0$.

C. Boundary conditions

The boundary conditions for a regular solution at the origin read

$$\mu(0) = 0 , \quad K(0) = 1 , \quad H(0) = 0 , \quad \xi'(0) = 0 \quad (17)$$

which have to be supplemented with our gauge fixing $\xi(0) = 0$. At infinity, finiteness of the mass and asymptotic flatness of the four-dimensional space-time requires:

$$A(\infty) = 1 , \quad K(\infty) = 0 , \quad H(\infty) = 1 , \quad \xi'(\infty) = 0 . \quad (18)$$

Note that the function $A(r)$ can in fact be rescaled at will and can be normalized according to $A(r_0) = 1$ at any point $r_0 \in [0, \infty]$

The boundary conditions at the regular horizon $x = x_h$ read:

$$N(x_h) = 0 \Rightarrow \tilde{\mu}(x_h) = \frac{x_h}{2} \quad (19)$$

with $A(x_h) < \infty$ and

$$(N'K')|_{x=x_h} = \left[\frac{K(K^2 - 1)}{x^2} + e^{-3\xi}H^2K \right] |_{x=x_h} , \quad (20)$$

$$(N'H')|_{x=x_h} = \left(\frac{2}{x^2}K^2H \right) |_{x=x_h} , \quad (21)$$

$$(N'\xi')|_{x=x_h} = \left\{ \frac{4}{3x^2} \alpha^2 \left[e^\xi \left(N(K')^2 + \frac{(K^2 - 1)^2}{2x^2} \right) - 2e^{-2\xi} \left(\frac{1}{2} N(H')^2 x^2 + H^2 K^2 \right) \right] \right\} |_{x=x_h} . \quad (22)$$

In this case as well we will choose the dilatation symmetry by demanding $\xi(x_h) = 0$.

One of the main results of our numerical analysis is that, in the case $\Lambda > 0$, the solution ends up at a cosmological horizon $x = x_c$ where $N(x_c) = 0$. The behaviour of the other metric fields for $x \rightarrow x_c$ suggest that the system of coordinates used is not appropriate for extending the solutions for $x > x_c$. As a consequence, it is impossible to impose a set of conditions of the type (22) at $x = x_c$. Appropriate boundary conditions have therefore to be enforced in the case $\Lambda > 0$. We will consider only the "realistic case" $\Lambda \ll 1$. In this case, at least the matter functions reaches their asymptotic values $K = 0, H = 1$ much before the cosmological horizon and can be imposed in the limit $x \rightarrow x_c$. The six other boundary conditions have to be imposed at the origin (or at the event horizon in the case of black strings).

III. REGULAR SOLUTION AT THE ORIGIN

A. Case $\Lambda = 0$

We solved the equations (11)-(13) subject to the conditions (17) and (18) (or (22) in the case of black holes) by numerical method for several values of α' . It turns out that, in the gauge chosen, solutions exist for a large domain of α' which we believe is infinite. Some characteristics are presented on Fig. 1; in particular the asymptotic value $\xi(\infty)$ and the charge of the dilaton say Q_d defined according to $\xi \approx \xi(\infty) + Q/x$.

B. Case $\Lambda > 0$

When the cosmological constant parameter is chosen to be positive, the function $N(x)$ develops (as expected) a zero at some finite value of the coordinate x , say $x = x_c$, this is called a cosmological horizon. Indeed, for $|x_c - x| \ll 1$, we find the behaviour

$$N(x) \sim N_c(x_c - x) \quad , \quad \xi(x) \sim \xi_i + \xi_c \sqrt{(x_c - x)} \quad , \quad A(x) = A_c(x_c - x)^{-a} \quad (23)$$

where $N_c, \xi_i, \xi_c, A_c, a$ are constants (with $a > 0$) depending on α and Λ . The best we can do is to integrate the equations on $x \in [0, x_c]$. For this purpose, we considered only values of

Λ such that the gauge and Higgs fields attain their asymptotic values for $x \ll x_c$. This is possible because $x_c \rightarrow \infty$ when $\Lambda \rightarrow 0$. So for sufficiently small values of the cosmological constant one is to impose $K(x_c) = 0, H(x_c) = 1$ as boundary conditions; all other conditions being supplemented at $x = 0$ (see (17)). The solution corresponding to $\alpha = 1, \Lambda = 0.0001$ is presented on Fig. 2; it corresponds to $x_c \approx 183$. The profiles of the corresponding solution for vanishing cosmological constant are also reported. We see how the cosmologic solution deviates from the asymptotically flat one at sufficiently large values of x . On the figure we can see that the function $a(r)$ develops a plateau at intermediate values between $r = 0$ and $r = r_c$. We found it convenient to normalize this function in such a way that it reaches the unit value on the plateau.

It is natural to study the evolution of the different parameters characterizing the solution at $r = r_c$ for different values of α' and of Λ . The analysis is not so easy because the singular point has to be approached closer and closer. However we performed such an analysis for the case $\Lambda = 0.0001$. The values of $x_c, \xi(x_c), N'(x_c)$ and a are reported of Fig.3. In particular, we observe that for $\alpha' > 1$ both values x_c and $\xi(x_c)$ increase for increasing α' . This can be related to the fact that the combination $e^{-\xi(x_c)}\Lambda$ determines an effective coupling constant which for instance become smaller for increasing α' . Explaining the fact that the value of the cosmological horizon is pushed towards infinity. Remember that, having fixed $\xi(0)$ as a boundary condition, we have no control on $\xi(x_c)$. The figure further shows that the values a and $N'(x_c)$ vary only a little with α' . The figure is limited to $\alpha' \leq 1.5$; the numerical analysis becomes more difficult with increasing α' , likely because both, the minimum N_m and the value $A(0)$ get smaller and smaller. The profiles of the metric functions available for $\alpha' = 1.5$ are presented on Fig.4 and illustrate our claim. By comparing Figs. 2 and 4, we can also appreciate the evolution of the dilaton function. It turns out that this function develops a local minimum in the neighborhood of the radius where N gets its minimum. We believe that Figs. 2-4 reflect the qualitative properties of a generic Λ .

It is a natural question to determine whether the singularity occurring at $x = x_c$ is essential or just a coordinate singularity. The computation of the Ricci scalar contains several terms of second order in the functions N and A , some of them are apparently singular but the complexity of this expression does not make easy to conclude. However, taking the trace of

the Einstein equations leads to

$$R = \frac{16\pi G}{2-d}T - \frac{2d}{2-d}\Lambda \quad , \quad T \equiv T_A^A \quad . \quad (24)$$

Further noticing that $T = -(1/2)\mathcal{L}_{ym}$ for five-dimensional gauge field theory, it turns out easy to evaluate R in terms of the lagrangian density \mathcal{L}_{ym} . Since the matter fields reach their expectation value for $x \rightarrow x_c$, it becomes obvious that R is finite for this limit. This suggests (although it is not a proof) that $x = x_c$ is a coordinate singularity and that the problem could be eliminated with more appropriate coordinates.

IV. BLACK STRINGS

A. Case $\Lambda = 0$

Before presenting our results, let us briefly recall how the pattern of solutions looks like [13, 14, 15] in the gauge $\xi(\infty) = 0$. The black string solutions exist in a limited domain of the α - x_h -plane. For fixed value of x_h several branches of solutions exist when varying α . This is very similar to the globally regular counterparts [11]. When α is fixed and x_h varied, always two branches of black string solutions exist. The lower branch extends from the corresponding globally regular solution with lowest energy up to a maximal value of $x_h(\alpha)$. The second branch either terminates at a finite x_h and joins the branch of Einstein-Maxwell-dilaton solutions or extends all the way back to $x_h = 0$, where it joins the branch of globally regular solutions with highest energy.

Again, the pattern of black string solutions looks completely different in the gauge $\xi(0) = 0$. For instance, the domain is not limited in the α' direction. For $\alpha' > \alpha'_c$ (with $\alpha'_c \sim 1.0$) we observe that there exist only one branch of solutions, indexed by x'_h and which bifurcates into an Einstein-Maxwell-dilaton solution at some maximal (α -depending) value $x'_{h,max}$. The sketch of the domain is presented in Fig. 5 ; this can be contrasted with Fig. 3 of [14] and on Fig. 6 different parameters characterizing the black string are plotted as functions of x'_h for $\alpha' = 0$. For $\alpha' < 1.0$ our numerical analysis indicates that the black strings solutions exist up to a maximal value as well but, then, another branch of solution exist while decreasing x'_h . Only on the second branch does the black string bifurcate into an abelian solution.

B. Case $\Lambda > 0$

Solutions behaving like black strings for $r \ll \infty$ and with a positive cosmological constant can be constructed numerically. It can be shown that they are plagued with the same singularity as their regular-at-the-origin counterparts (see Sect. IV.B). The profile of such a solution corresponding to $\alpha = 1, x_h = 0.3$ is presented on Fig. 7.

V. $\Lambda > 0$ BLACK STRINGS - VACUUM CASE

After observing the recurrent pathologies of the non abelian solutions in the presence of a positive cosmological constant, we turned out to the case where matter is absent. In the case of a negative cosmological constant, the solutions have been constructed in [20].

If we set $\alpha = 0$ the matter fields equations above are trivially satisfied by $K = 1, H = 0$ and we are left with a system of three Einstein equations for $m(r), A(r), \xi(r)$. The cosmological constant Λ can be absorbed by means of a translation of ξ . Solving these equations with the regular conditions at the origin (or the ones of an event horizon at $r = r_c$) leads to the same pathologies as observed in the previous section. We therefore integrated the equations by imposing the regular conditions at the cosmological horizon and then integrating from $r = r_c$. Because of the scale invariance of the system, the value r_c can be chosen arbitrarily and we set $r_c = 10$. An extra translation of the function ξ allows one to fix the boundary condition "à la Cauchy", i.e.

$$m(r_c) = \frac{r_c}{2} \quad , \quad A(r_c) = 1 \quad , \quad \xi(r_c) = 0 \quad , \quad \xi'(r_c) = -\frac{2\Lambda r_c}{3(2 - r_c^2)} \quad (25)$$

where the last condition ensures the regularity at the horizon.

Integrating the equations inside the horizon, our numerical analysis indicates that the solution becomes singular at the approach of the origin. For instance we find :

$$m(r) \sim M_0 r^\omega \quad , \quad \xi(r) \sim \xi_0 + c \log(r) \quad , \quad A(r) \sim A_0 r^{3c^2/4} \quad (26)$$

where M_0, A_0, c and ω are constants. Such a behaviour is confirmed by the asymptotic analysis of the equations. The power ω and the other coefficients are determined numerically. In the case $\Lambda = 0.1$ we find $M_0 \approx -28.057$, $\omega \approx -0.7585$, $\xi_0 \approx -2.1579$, and $c \approx 1.0056$. The profiles of the functions are represented on Fig.1. Comparaison with the non Abelian

solutions discussed in the previous section shows how much the gauge fields regularize the solutions at the origin.

Integrating the equations outside the sphere determined by the cosmological horizon leads to a behaviour similar to the one above. The corresponding parameters are $\omega \approx 2.31, c \approx 0.67$. The asymptotic solution is clearly not DeSitter. We see that the occurrence of a positive cosmological constant has important effects on the black string solutions and leads to completely different properties than in the case of a negative cosmological constant [20]. Notice that these solutions presented here are different from the analytical solutions reported in Sect. 4 of [18] which have no cosmological horizon. A systematic study of vacuum black string solutions with positive cosmological constant and in arbitrary dimensions will be presented elsewhere [21].

VI. SUMMARY

We investigated the most natural extension of the gravitating monopole living in a five-dimensional space-time endowed with a cosmological constant. For technical reasons, we were forced to reconstruct the non-abelian black strings in another gauge where the dilaton is imposed to be zero at the origin (or at the regular event horizon x_h in the case of black strings). Although equivalent to the case of the $\xi(\infty) = 0$ -gauge (chosen e.g. in [13, 14]), the pattern of black strings solutions looks completely different. We have characterized the domain of solutions in this new frame.

We further found that supplementing the five dimensional space-time by a positive cosmological constant leads to a cosmological horizon which constitutes an apparent singularity of the solution. The situation, however contrasts to the four dimensional case where all functions remain analytic when the cosmological horizon is approached [4]; this feature is typically due to the occurrence of a dilaton field. Considering the same equations in absence of gauge and Higgs fields, our results strongly suggest that the presence of a positive cosmological constant does not deform smoothly the uniform black strings and leads to solution which present a singularity at the origin and whose asymptotic behaviour is not DeSitter.

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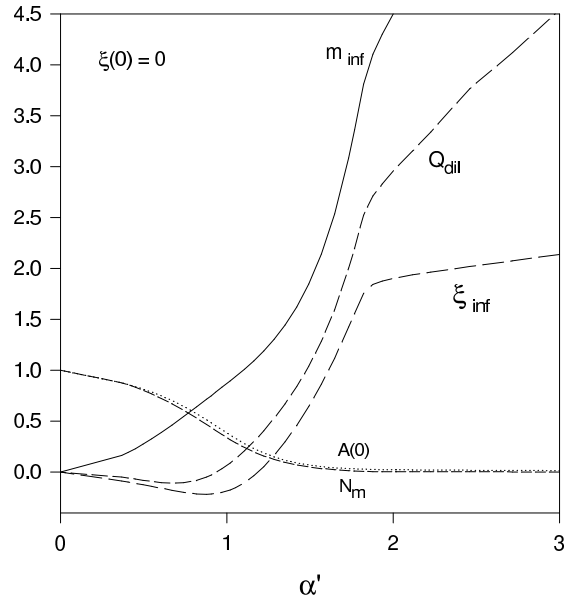


FIG. 1: The values of the mass, dilaton charge, $A(0)$, N_{\min} and $\xi(\infty)$ are plotted as functions of α_c for the solutions with $\xi(0) = 0$

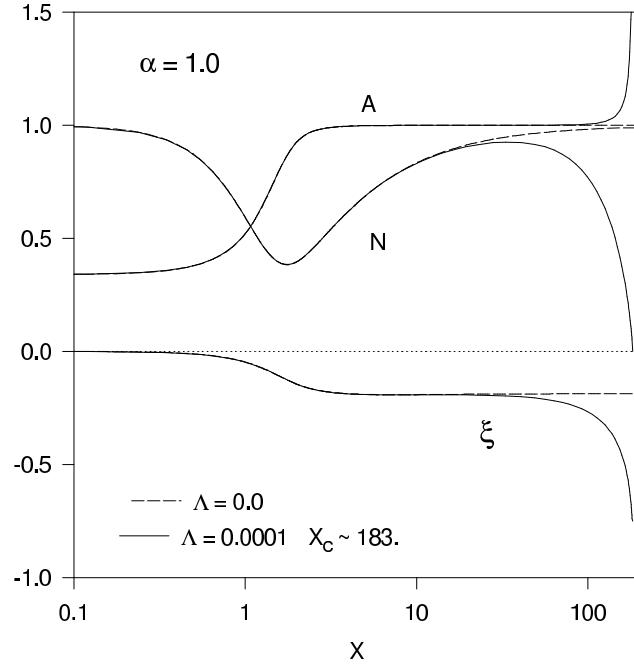


FIG. 2: The profile of the metric functions corresponding to $\alpha_c = 1$ for $\Lambda = 0.0$ and $\Lambda = 0.0001$.

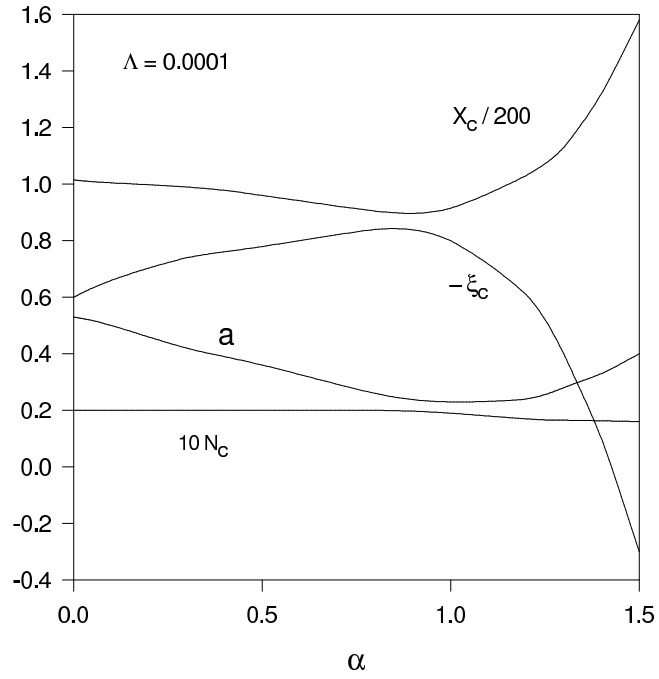


FIG. 3: The evolution of the parameters at the horizon are shown as functions of α for $\Lambda = 0.0001$.

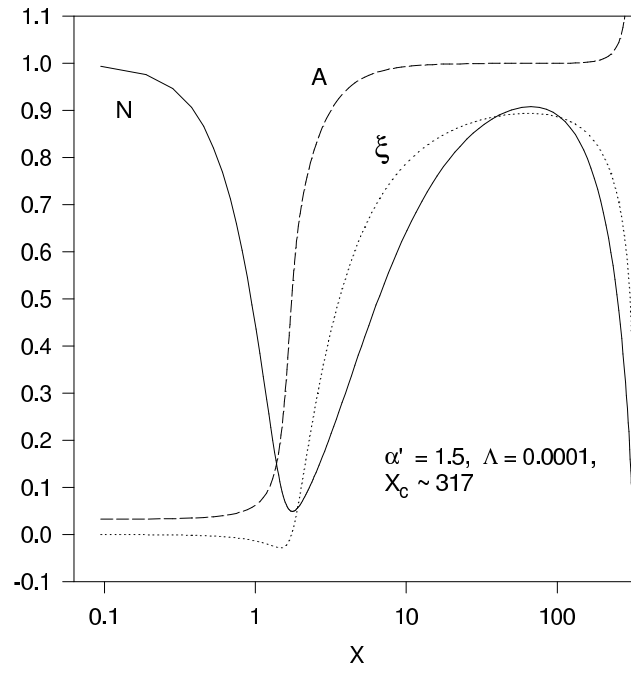


FIG. 4: Idem fig.2 for $\alpha = 1.5$.

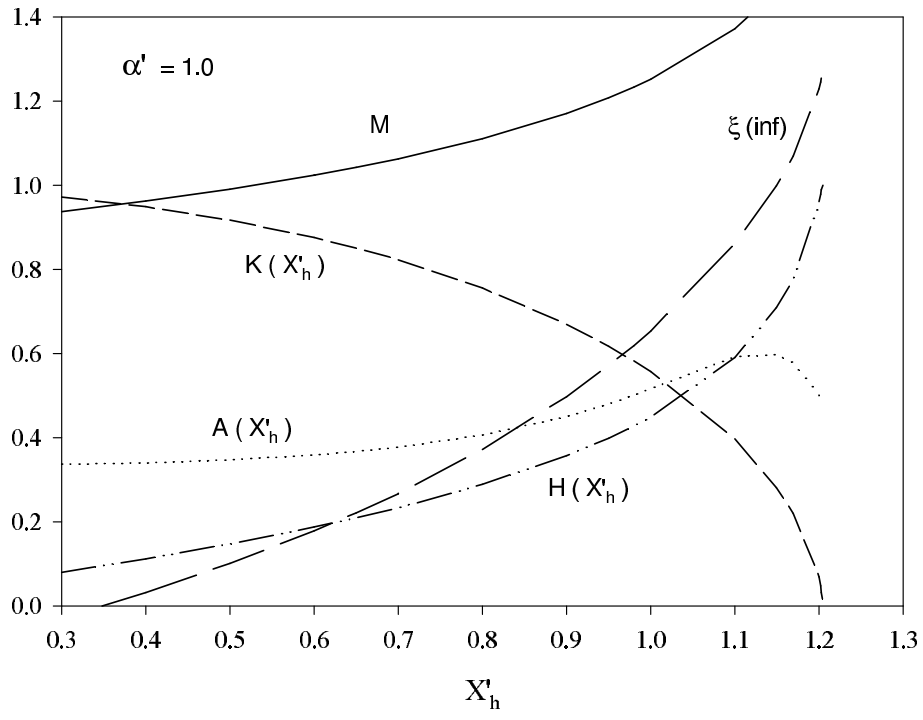


Fig 5 The parameters characterizing the black string as functions of the horizon for $\alpha' = 1$

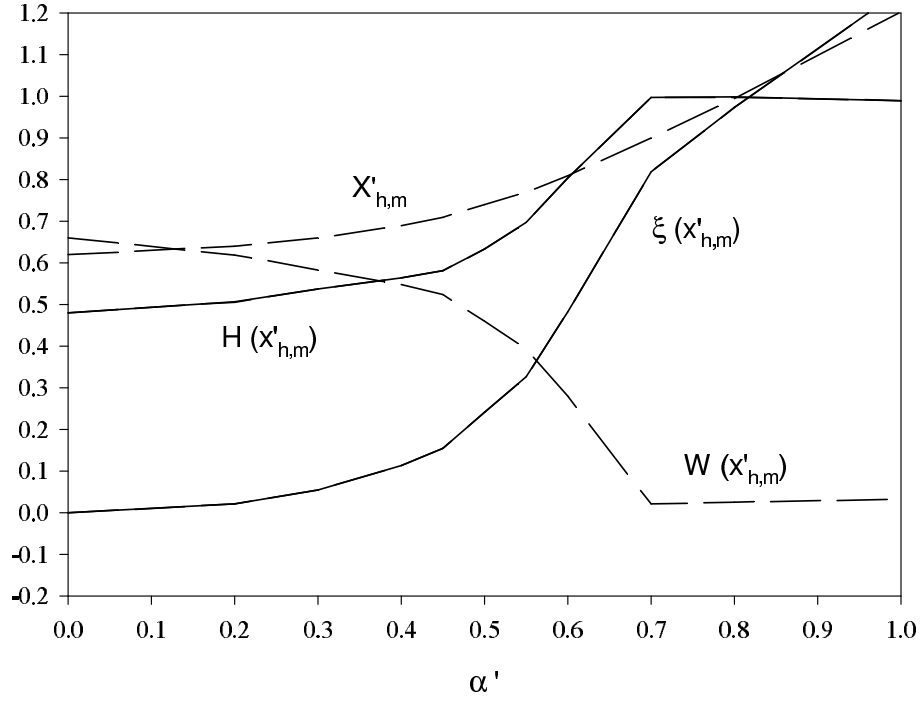


Fig 6 The maximal value of the horizon $x'_{h,max}$ is plotted as a function of α' a few parameters related to the transition are supplemented

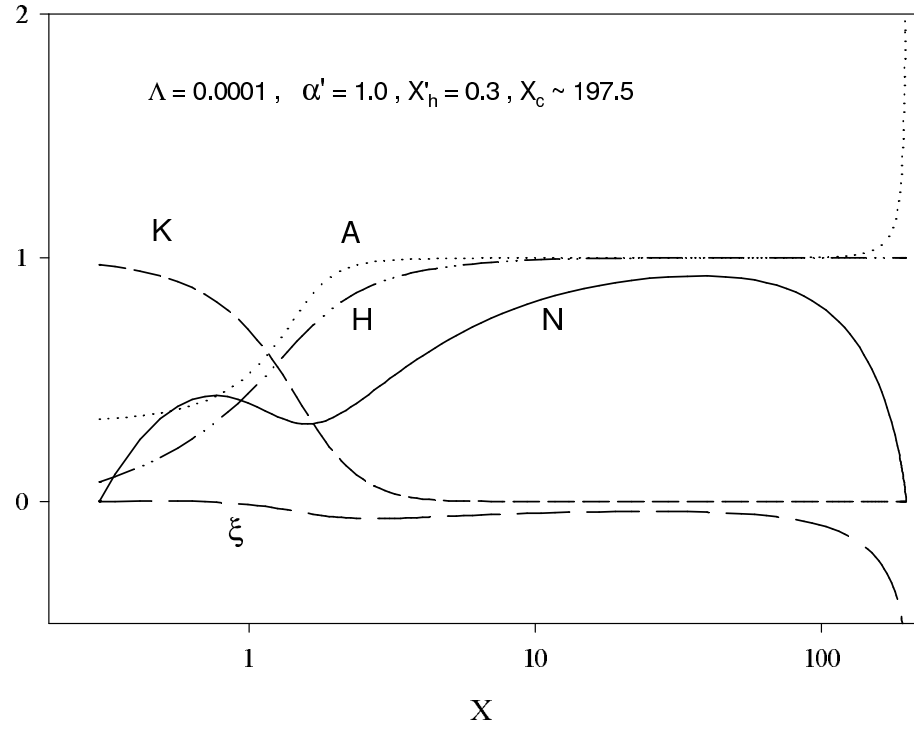


Fig 7 The profiles of a black string with cosmological constant $\Lambda = 0.0001$

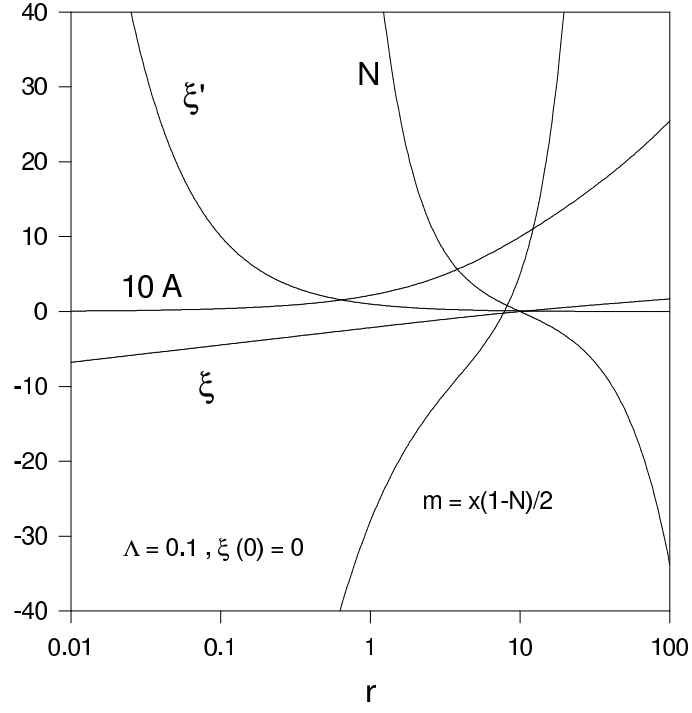


Fig 8 The values of the mass, dilaton charge, $A(0)$, N_{min} and $\xi(\infty)$ are plotted as functions of α_c for the solutions with $\xi(0) = 0$